Local Receptive Fields Based Extreme Learning Machine

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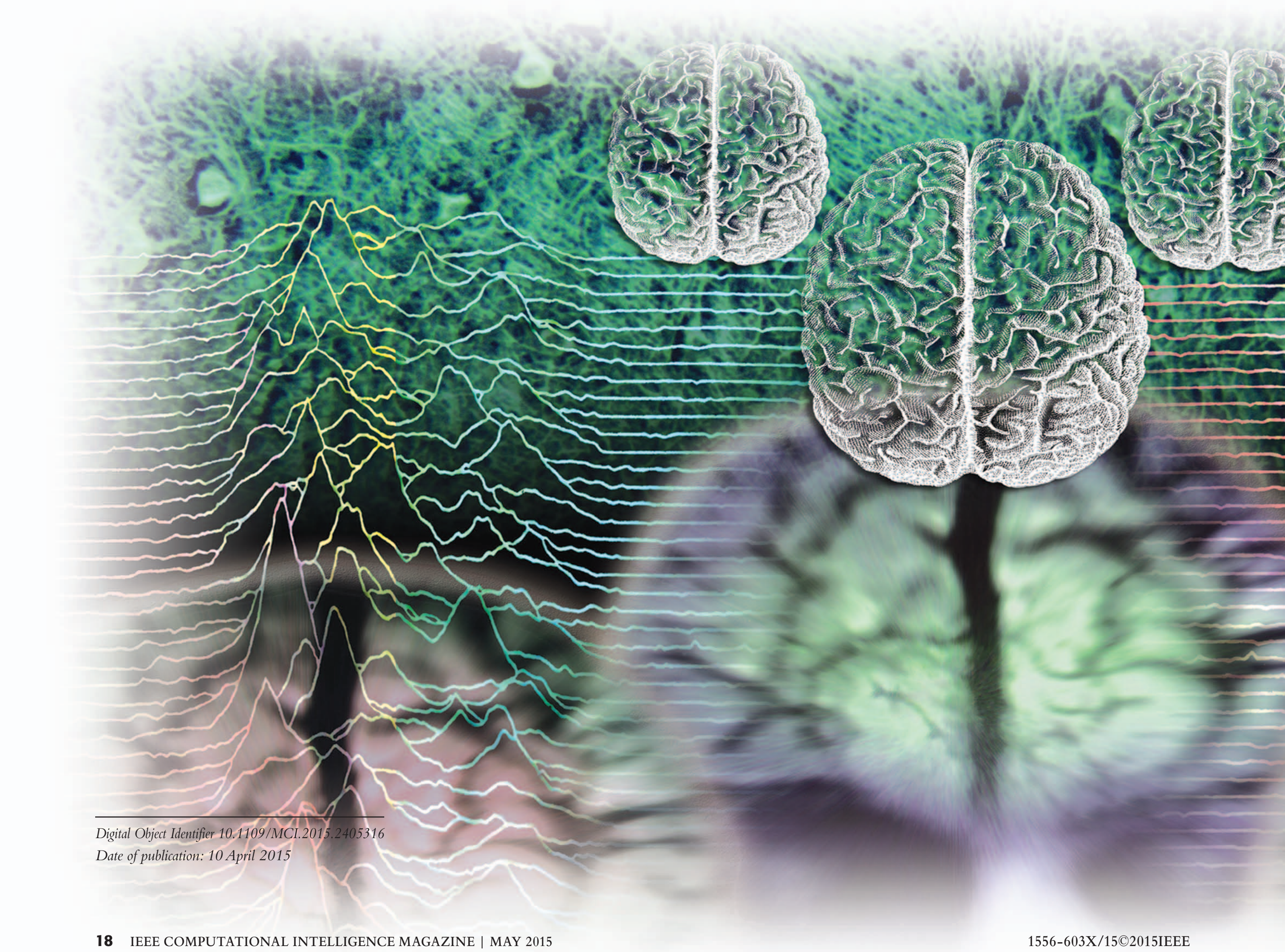
Abstract—Extreme learning machine (ELM), which was originally proposed for "generalized" single-hidden layer feedforward neural networks (SLFNs), provides efficient unified learning solutions for the applications of feature learning, clustering, regression and classification. Different from the common understanding and tenet that hidden neurons of neural networks need to be iteratively adjusted during training stage, ELM theories show that hidden neurons are important but need not be iteratively tuned. In fact, all the parameters of hidden nodes can be independent of training samples and randomly generated according to any continuous probability



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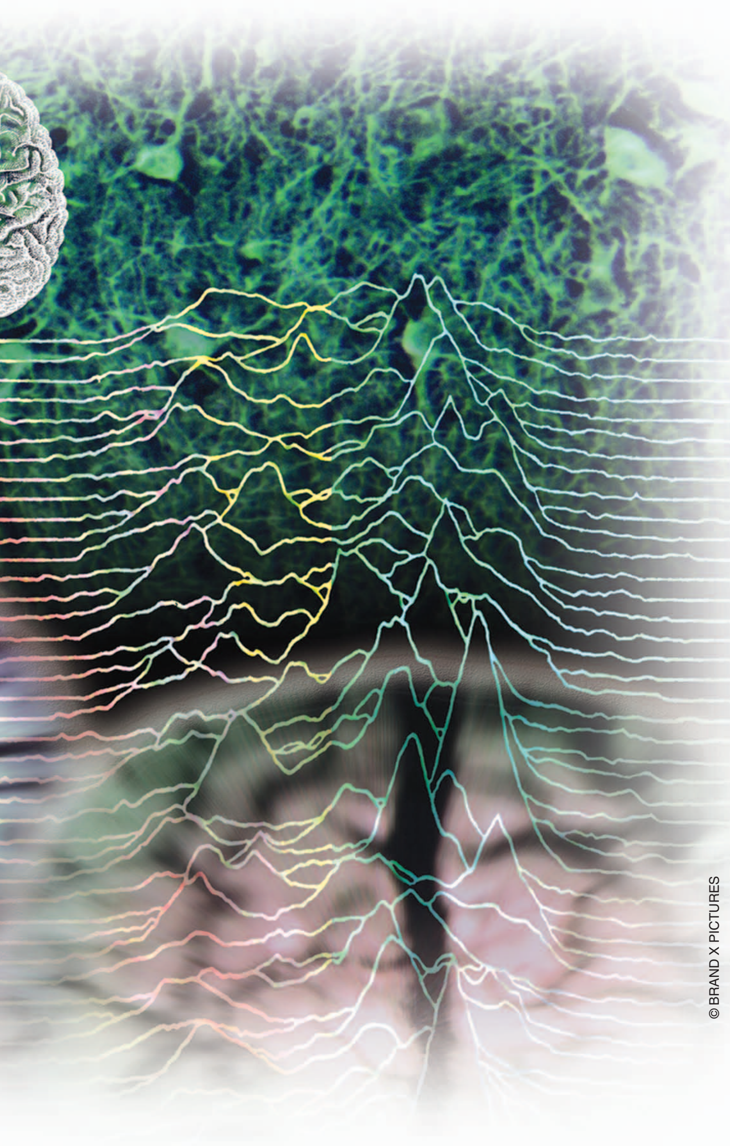
distribution. And the obtained ELM networks satisfy universal approximation and classification capability. The fully connected ELM architecture has been extensively studied. However, ELM with local connections has not attracted much research attention yet. This paper studies the general architecture of locally connected ELM, showing that: 1) ELM theories are naturally valid for local connections, thus introducing local receptive fields to the input layer; 2) each hidden node in ELM can be a combination of several hidden nodes (a subnetwork), which is also consistent with ELM theories. ELM theories may shed a light on the research of different local receptive fields including true biological receptive fields of which the exact shapes and formula may be unknown to human beings. As a specific example of such general architectures, random convolutional nodes and a pooling structure are implemented in this paper. Experimental results on the NORB dataset, a benchmark for object recognition, show that compared with conventional deep learning solutions, the proposed local receptive fields based ELM (ELM-LRF) reduces the error rate from 6.5% to 2.7% and increases the learning speed up to 200 times.



I. Introduction

Machine learning and big data analysis have recently drawn great attentions from researchers in different disciplinaries [1]–[5]. To the best of our knowledge, the success of machine learning relies on three key factors: powerful computing environments, rich and dynamic data, and efficient learning algorithms. More importantly, efficient machine learning algorithms are highly demanded in the big data applications.

Most conventional training methods designed for neural networks such as back-propagation (BP) algorithm [6] involve numerous gradient-descent searching steps and suffer from troubles including slow convergence rate, local minima, intensive human intervention, etc. Extreme learning machine (ELM) [7]–[13] aims to overcome these drawbacks and limitations faced by conventional learning theories and techniques. ELM provides an efficient and unified learning framework to "generalized" single-hidden layer feedforward neural networks (SLFNs) including but not limited to sigmoid networks, RBF networks, threshold networks, trigonometric networks, fuzzy inference systems, fully complex neural networks, high-order networks, ridge polynomial networks, wavelet networks, Fourier series, etc [9], [10], [13]. It presents competitive accuracy with superb efficiency in many different applications such as biomedical analysis [14], [15], chemical process [16], system modeling [17], [18], power systems [19], action recognition [20], hyperspectral images [21], etc. For example, ELM auto-encoder [5] outperforms state-of-the-art deep learning methods in MNIST dataset with significantly improved learning accuracy while dramatically reducing the training time from almost one day to several minutes.

In almost all ELM implementations realized in the past years, hidden nodes are fully connected to the input nodes. Fully connected ELMs are efficient in implementation and produce good generalization performance in many applications. However, some applications such as image processing and speech recognition may include strong local correlations, and it is reasonably expected that the corresponding neural networks have local connections instead of full ones so that local correlations could be learned. It is known that local receptive fields actually lie in the retina module of biological learning system, which help consider local correlations of the input images. This paper aims to address the open problem: *Can local receptive fields be implemented in ELM?* ELM theories [9]–[11] prove that hidden nodes can be randomly generated according to *any continuous probability distribution*. Naturally speaking, in some applications hidden nodes can be generated according to some continuous probability distributions which are denser around some input nodes while sparser farther away. In this sense, ELM theories are actually valid for local receptive fields, which have not yet been studied in the literature.

According to ELM theories [9]–[11], local receptive fields can generally be implemented in different "shapes" as long

Unlike the common tenet that hidden neurons need to be iteratively adjusted during training stage, ELM theories show that hidden neurons are important but need not be iteratively tuned in many types of neural networks.

as they are continuous and local-wise. However, from application point of view, such "shapes" could be application type dependent. Inspired by convolutional neural networks (CNNs) [22], one of such local receptive fields can be implemented by randomly generating convolutional hidden nodes and the universal approximation capability of such ELM can still be preserved. Although local receptive fields based ELM and CNN are similar in the sense of local connections, they have different properties as well:

1) Local receptive fields: CNN uses convolutional hidden nodes for local receptive fields. To the best knowledge of ours, except for the biological inspiration, *there is no theoretical answer in literature on why convolutional nodes can be used*. However, *according to ELM theories, ELM can flexibly use different types of local receptive fields as long as they are randomly generated according to any continuous probability distribution*. The random convolutional hidden node is one type of those nodes with local receptive fields that can be used in ELM. This paper actually builds the theoretical relationship between random hidden nodes and local receptive fields. Especially from theoretical point of view, it addresses why random convolutional nodes can be used as an implementation of local receptive fields. On the other hand, ELM works for wide types of hidden nodes.
2) Training: In general, similar to most conventional neural networks, all the hidden nodes in CNN need to be adjusted and BP learning methods are used in the tuning. Thus, CNN learning faces the problems inherited from BP algorithm such as local minima, time consuming and intensive human intervention. In contrast, ELM with hidden nodes functioning within local receptive fields keeps the essence of ELM: the hidden nodes are randomly generated and the output weights are analytically calculated with minimum norm of output weights constraints, which provides a deterministic solution that is simpler, stable, and more efficient.

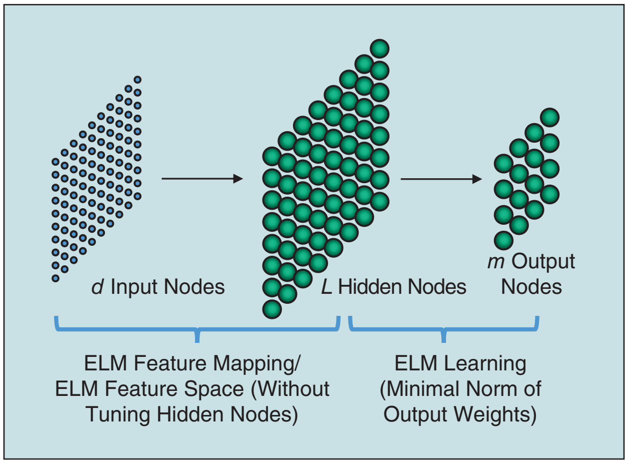


FIGURE 1 Two-stage ELM architecture: ELM feature mapping and ELM learning. ELM feature mapping is usually formed by single or multi-type of random hidden neurons which are independent of training data. ELM learning focuses on calculating the output weights in some form which may be application dependent (e.g., feature learning, clustering, regression/classification, etc). Closed-form solutions have been used in many applications.

II. Reviews of ELM, CNN and HTM

ELM is well-known and widely used for its high efficiency and superior accuracy [8], [12], [13], [23]. CNN is notably suitable for image processing and many other tasks which need to consider local correlations [24]. Hierarchical temporal memory (HTM) intends to unravel the mystery of intelligence by providing a memory-based prediction framework [25]. This section gives a brief review of ELM, CNN and HTM. Detailed reviews of ELM can be found in Huang *et al.* [13], [26].

A. Extreme Learning Machine (ELM)

ELM was initially proposed for single-hidden layer feedforward neural networks (SLFNs) [7]–[9] and then extended to the "generalized" SLFNs with wide types of hidden neurons [10]–[12]. Different from the common understanding of neural networks, ELM theories [9]–[11] show that although hidden neurons play critical roles, they need not be adjusted. Neural networks can achieve learning capability without iteratively tuning hidden neurons as long as the activation functions of these hidden neurons are nonlinear piecewise continuous including but not limited to biological neurons. The output function of biological neurons may not be explicitly expressed with concrete closed-form formula and different neurons may have different output functions. However, the output functions of most biological neurons are nonlinear piecewise continuous which have been considered by ELM theories. Both Random Vector Functional-Link (RVFL) [27]–[32] and QuickNet [33]–[36] use direct links between the input layer and the output layer while using a specific ELM hidden layer (random sigmoid hidden layer). (Readers can refer to [13] for detailed analysis of the relationship and differences between ELM and RVFL/QuickNet).

As shown in Fig. 1, ELM first transforms the input into hidden layer through ELM feature mapping. Then the output is generated through ELM learning, which could be classification, regression, clustering, etc. [13], [37].

1) ELM feature mapping: The output function of ELM for generalized SLFNs is:

$$f(\mathbf{x}) = \sum_{i=1}^{L} \boldsymbol{\beta}_i h_i(\mathbf{x}) = \mathbf{h}(\mathbf{x})\boldsymbol{\beta} \tag{1}$$

where $\boldsymbol{\beta} = [\boldsymbol{\beta}_1, \cdots, \boldsymbol{\beta}_L]^T$ is the vector of the output weights between the hidden layer with L nodes to the output layer with

$m \geq 1$ nodes; and $\mathbf{h}(\mathbf{x}) = [h_1(\mathbf{x}), \cdots, h_L(\mathbf{x})]$ is the output (row) vector of the hidden layer. *Different activation functions may be used in different hidden neurons.* In particular, in real applications $h_i(\mathbf{x})$ can be

Single network architecture may be capable of feature learning, clustering, regression and classification.

$$h_i(\mathbf{x}) = G(\mathbf{a}_i, b_i, \mathbf{x}), \mathbf{a}_i \in \mathbf{R}^d, b_i \in R \tag{2}$$

where $G(\mathbf{a}, b, \mathbf{x})$ is a nonlinear piecewise continuous function and $(\mathbf{a}_i, b_i)$ are the i-th hidden node parameters. Some commonly used activation functions are:

i) Sigmoid function:

$$G(\mathbf{a}, b, \mathbf{x}) = \frac{1}{1 + \exp(-(\mathbf{a} \cdot \mathbf{x} + b))} \tag{3}$$

ii) Fourier function [9], [38]:

$$G(\mathbf{a}, b, \mathbf{x}) = \sin(\mathbf{a} \cdot \mathbf{x} + b) \tag{4}$$

iii) Hardlimit function [9], [39]:

$$G(\mathbf{a}, b, \mathbf{x}) = \begin{cases} 1, & \text{if } \mathbf{a} \cdot \mathbf{x} - b \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

iv) Gaussian function [9], [12]:

$$G(\mathbf{a}, b, \mathbf{x}) = \exp(-b\|\mathbf{x} - \mathbf{a}\|^2) \tag{6}$$

v) Multiquadrics function [9], [12]:

$$G(\mathbf{a}, b, \mathbf{x}) = (\|\mathbf{x} - \mathbf{a}\|^2 + b^2)^{1/2} \tag{7}$$

To the best knowledge of ours, from 1950s to early this century, almost all the learning theories show that the hidden neurons of standard feedforward neural networks need to be tuned. *In contrast, different from conventional artificial neural networks theories, ELM theories show that hidden neurons need not be tuned.* One of its implementation is random hidden neurons. A neuron is called a random neuron if all its parameters (e.g., $(\mathbf{a}, b)$ in its output function $G(\mathbf{a}, b, \mathbf{x})$) are randomly generated based on a continuous probability distribution. $\mathbf{h}(\mathbf{x})$ actually maps the data from the d-dimensional input space to the L-dimensional hidden layer random feature space (also called *ELM feature space*) where the hidden node parameters are randomly generated according to any continuous probability distribution, and thus, $\mathbf{h}(\mathbf{x})$ is indeed a random feature mapping (also called *ELM feature mapping*).

2) *ELM learning:* From the learning point of view, unlike traditional learning algorithms [6], [33], [40], ELM theories emphasize that *the hidden neurons need not be adjusted*, and ELM solutions aim to *simultaneously reach the smallest training error and the smallest norm of output weights* [7], [8], [12], [13]:

$$\text{Minimize:} \|\boldsymbol{\beta}\|_p^{\sigma_1} + C\|\mathbf{H}\boldsymbol{\beta} - \mathbf{T}\|_q^{\sigma_2} \tag{8}$$

where $\sigma_1 > 0, \sigma_2 > 0, p, q = 0, 1, 2, 3, 4, \cdots, +\infty$ (p, q may not always be integer, e.g., $p, q = \frac{1}{2}, \cdots$), and C is the parameter controlling the trade-off between these two terms. Given a set of training samples $(\mathbf{x}_i, \mathbf{t}_i), i = 1, \cdots, N, \mathbf{H}$ is the hidden layer output matrix (*nonlinear transformed randomized matrix*):

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}(\mathbf{x}_1) \\ \vdots \\ \mathbf{h}(\mathbf{x}_N) \end{bmatrix} = \begin{bmatrix} h_1(\mathbf{x}_1) & \cdots & h_L(\mathbf{x}_1) \\ \vdots & \vdots & \vdots \\ h_1(\mathbf{x}_N) & \cdots & h_L(\mathbf{x}_N) \end{bmatrix} \tag{9}$$

and $\mathbf{T}$ is the training data target matrix:

$$\mathbf{T} = \begin{bmatrix} \mathbf{t}_1^T \\ \vdots \\ \mathbf{t}_N^T \end{bmatrix} = \begin{bmatrix} t_{11} & \cdots & t_{1m} \\ \vdots & \vdots & \vdots \\ t_{N1} & \cdots & t_{Nm} \end{bmatrix} \tag{10}$$

Numerous efficient methods can be used to calculate the output weights $\boldsymbol{\beta}$ including but not limited to orthogonal projection methods, iterative methods [23], [41], [42], and singular value decomposition (SVD) [43]. A popular and efficient closed-form solution for ELM with $\sigma_1 = \sigma_2 = p = q = 2$ [12], [42], [44] is:

$$\beta = \begin{cases} \mathbf{H}^T\left(\frac{\mathbf{I}}{C} + \mathbf{H}\mathbf{H}^T\right)^{-1}\mathbf{T}, & \text{if } N \leq L \\ \left(\frac{\mathbf{I}}{C} + \mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{T}, & \text{if } N > L \end{cases} \tag{11}$$

ELM satisfies both universal approximation and classification capability:

Theorem 2.1: Universal approximation capability [9]–[11]: Given any nonconstant piecewise continuous function as the activation function, if tuning the parameters of hidden neurons could make SLFNs approximate any target function $f(\mathbf{x})$, then the sequence $\{h_i(\mathbf{x})\}_{i=1}^L$ can be randomly generated according to any continuous distribution probability, and $\lim_{L \to \infty}\left\|\sum_{i=1}^L \boldsymbol{\beta}_i h_i(\mathbf{x}) - f(\mathbf{x})\right\| = 0$ holds with probability 1 with appropriate output weights $\boldsymbol{\beta}$.

Theorem 2.2: Classification capability [12]: Given any nonconstant piecewise continuous function as the activation function, if tuning the parameters of hidden neurons could make SLFNs approximate any target function $f(\mathbf{x})$, then SLFNs with random hidden layer mapping $\mathbf{h}(\mathbf{x})$ can separate arbitrary disjoint regions of any shapes.

Remark: For the sake of completeness, it may be better to clarify the relationships and differences between ELM and other related works: QuickNet [33]–[36], Schmidt, *et al.* [40], and RVFL [27]–[32]:

1) Network architectures: ELM is proposed for "generalized" single-layer feedforward networks:

$$f_L(\mathbf{x}) = \sum_{i=1}^L \boldsymbol{\beta}_i G(\mathbf{a}_i, b_i, \mathbf{x}) \tag{12}$$

As ELM has universal approximation capability for a wide type of nonlinear piecewise continuous function $G(\mathbf{a}, b, \mathbf{x})$, it does not need any bias in the output layer. Different from ELM but similar to SVM [45], the feedforward neural network with random weights proposed in Schmidt, *et al.* [40] requires a bias in the output node in order to absorb the system error as its universal approximation capability was not proved when proposed:

Hidden nodes can be generated according to some continuous probability distributions which are denser around some input nodes while sparser farther away, which naturally results in local receptive fields.

$$f_L(\mathbf{x}) = \sum_{i=1}^{L} \beta_i g(\mathbf{a}_i \cdot x + b_i) + b \tag{13}$$

If the output neuron bias is considered as a bias neuron in the hidden layer as done in most conventional neural networks, the hidden layer output matrix for [40] will be

$$\mathbf{H}_{\text{Schmidt, } et\ al. (1992)} = [\mathbf{H}_{\text{ELM for sigmoid basis}} \quad \mathbf{1}_{N \times m}] \tag{14}$$

where $\mathbf{H}_{\text{ELM for sigmoid basis}}$ is a specific ELM hidden layer output matrix (9) with sigmoid basis, and $\mathbf{1}_{N\times m}$ is a $N \times m$ matrix with constant element 1. Although it seems that bias b is just a simple parameter, its role has drawn researchers' attention [13], [46], [47], and indeed from optimization point of view, to introduce a bias will result in a suboptimal solution [12], [13]. Both QuickNet and RVFL have the direct link between the input node and the output node:

$$f_L(\mathbf{x}) = \sum_{i=1}^{L} \beta_i g(\mathbf{a}_i \cdot x + b_i) + \alpha \mathbf{x} \tag{15}$$

Different from Schmidt, *et al.*'s model and RVFL in which only sigmoid or RBF type of hidden nodes are used, each hidden node of ELM G($\mathbf{a}$, b, $\mathbf{x}$) can even be a super node consisting of a subnetwork [10]. (Refer to Section III for details.)

2) Objective functions: Similar to conventional learning algorithms Schmidt, *et al.*'s model and RVFL focus on minimizing the training errors. Based on neural network generalization performance theory [48], ELM focuses on minimizing both training errors and the structural risk errors [7], [8], [12], [13] (as shown in equation (8)). As analyzed in Huang [13], compared to ELM, Schmidt, *et al.*'s model and RVFL actually provide suboptimal solutions.
3) Learning algorithms: Schmidt, *et al.*'s model and RVFL mainly focus on regression while ELM is efficient for feature learning, clustering, regression and classification. ELM is efficient for auto encoder as well [5]. However, when RVFL is used for auto encoder, the weights of the direct link between its input layer to its output layer will become one and the weights of the links between its hidden layer to its output layer will become zero, thus, RVFL will lose learning capability in auto encoder cases too.
4) Learning theories: The universal approximation and classification capability of Schmidt, *et al.*'s model have not been proved before ELM theories are proposed [9]–[11]. Its universal approximation capability will become apparent when ELM theory is applied. Igelnik and Pao [32] actually only prove RVFL's universal approximation capability when semi-random hidden nodes are used, that is, the input weights $\mathbf{a}_i$ are randomly generated while the hidden node biases b_i are calculated based on the training samples $\mathbf{x}_i$ and the input weights $\mathbf{a}_i$. In contrast to RVFL theories for semi-randomness, ELM theories show that i) Generally speaking, all the hidden node parameters can be randomly generated as long as the activation function is nonlinear piecewise continuous; ii) all the hidden nodes can be not only independent from training samples but also independent from each other; iii) ELM theories are valid for but not limited to sigmoid networks, RBF networks, threshold networks, trigonometric networks, fuzzy inference systems, fully complex neural networks, high-order networks, ridge polynomial networks, wavelet networks, Fourier series, and biological neurons whose modeling/shapes may be unknown, etc [9], [10], [13].

B. Convolutional Neural Network (CNN)

Convolutional neural network (CNN) is a variant of multi-layer feedforward neural networks (or called multi-layer perceptrons (MLPs)) and was originally biologically inspired by visual cortex [49]. CNN is often used in image recognition applications. It consists of two basic operations: convolution and pooling. Convolutional and pooling layers are usually arranged alternately in the network, until obtaining the high-level features on which classification is performed. In addition, several feature maps may exist in each convolutional layer and the weights of convolutional nodes in the same map are shared. This setting enables the network to learn different features while keeping the number of parameters tractable.

Compared with traditional methods, CNN is less task specific since it implicitly learns the feature extractors rather than specifically designing them. The training of CNN usually relies on the conventional BP method, and thus CNN requires iterative updates and suffers from trivial issues associated with BP.

1) Convolution: For a convolutional layer, regardless of which layer it is in the whole network, suppose that the size of the layer before is $d \times d$ and the receptive field is $r \times r$. Valid convolution is adopted in CNN, where y and x respectively denote the values of convolutional layer and the layer before:

$$\begin{aligned} y_{i,j} &= g\left(\sum_{m=1}^{r}\sum_{n=1}^{r} x_{i+m-1,j+n-1} \cdot w_{m,n} + b\right) \\ i,j &= 1, \cdots, (d-r+1) \end{aligned} \tag{16}$$

where g is a nonlinear function.

2) Pooling: Pooling is followed after convolution to reduce the dimensionality of features and introduce translational invariance into the CNN network. Different pooling methods are performed over local areas, including averaging and maxpooling [50].

C. Hierarchical Temporal Memory (HTM)

HTM is a general hierarchical memory-based prediction framework inspired from neocortex [25]. It argues that the inputs into the neocortex are essentially alike and contain both spatial and temporal information. Firstly, the inputs arrive the lowest level, primary sensory area, of the hierarchical framework. Then, they pass through the association areas, where spatial and temporal patterns are combined. Finally, complex and high-level abstraction of the patterns are generated and stored in the memory. HTM shows that hierarchical areas and different parts in the neocortex are extraordinarily identical. And the intelligence could be realized through this hierarchical structure, solving different problems, including vision, hearing, touch, etc.

ELM theories may shed a light on the research of different local receptive fields including true biological receptive fields of which the exact shapes and formula may be unknown to human beings.

III. Local Receptive Fields Based Extreme Learning Machine

In this section, we propose the local receptive fields based ELM (ELM-LRF) as a generic ELM architecture to solve image processing and similar problems in which different density of connections may be requested. The connections between input and hidden nodes are sparse and bounded by corresponding receptive fields, which are sampled by any continuous probability distribution. Additionally, combinatorial nodes are used to provide translational invariance to the network by combining several hidden nodes together. It involves no gradient-descent steps and the training is remarkably efficient.

A. Hidden Nodes in Full and Local Connections

ELM theories [9]–[11] proved that hidden nodes can be generated randomly according to any probability distribution. In essence, the randomness is two-folded:

1) Different density of connections between input and hidden nodes are randomly sampled due to different types of probability distributions used in applications.
2) The weights between input and hidden nodes are also randomly generated.

Fully connected hidden nodes, as shown in Fig. 2(a), have been extensively studied. Constructed from them, ELM achieves state-of-the-art performance for plenty of applications, such as remote sensing [51], time series analysis [52], text classification [53], action recognition [20], etc.

However, these works focus only on random weights, while ignoring the attribute of random connections. For natural images and languages, the strong local correlations may make the full connections less appropriate. To overcome the problem, we propose to use hidden nodes which are locally connected with the input ones. According to ELM theories [9]–[11] the connections are randomly sampled based on certain probability distributions, which are denser around some input nodes while sparser farther away.

B. Local Receptive Fields Based ELM

A locally dense connection example of ELM-LRF is given in Fig. 2(b) in which the connections between input layer and hidden node i are randomly generated according to some continuous probability distribution, and some receptive field is generated by this random connections. The connections are dense around some input nodes and sparse farther away (yet do not completely disappear in this example).

Solid biological evidence also justifies the local receptive fields, which shows that each cell in the visual cortex is only sensitive to a corresponding sub-region of the retina (input layer) [54]–[56]. ELM-LRF learns the local structures and generates more meaningful representations at the hidden layer when dealing with image processing and similar tasks.

C. Combinatorial Node

Local receptive fields may have different forms in biological learning, and they may be generated in different ways using machine learning techniques. One of the methods is the combinatorial node suggested in ELM theory [10]. ELM shows that a hidden node in ELM could be a combination of several hidden nodes or a subnetwork of nodes. For example, local receptive fields shown in Fig. 3 could be considered as a case where combinatorial node i is formed by a subnetwork (as grouped by the circle). The output of this subnetwork is actually a summation (which need not be linear) of the output of three hidden nodes linking to corresponding local receptive fields of the input layer.

Thus, the feature generated at one location (one particular hidden node) tends to be useful at different locations.

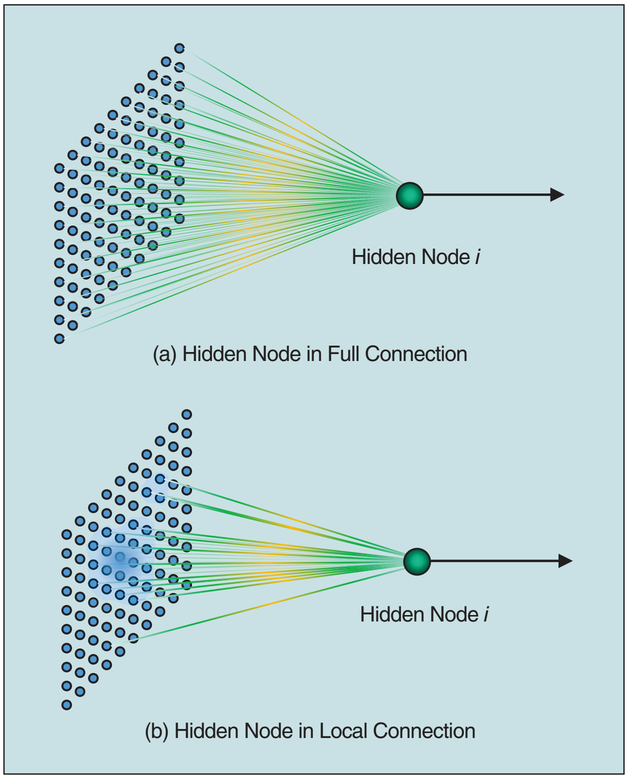


FIGURE 2 ELM hidden node in full and local connections: Random connections generated due to various continuous distributions.

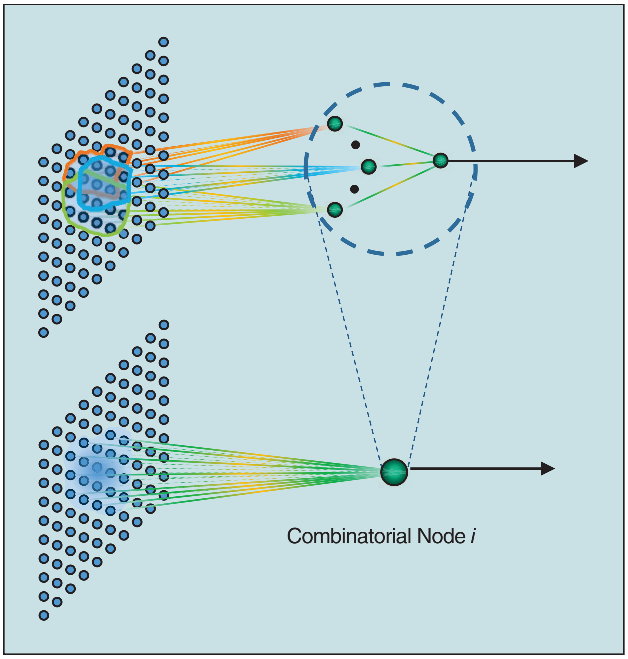


FIGURE 3 The combinatorial node of ELM [10]: a hidden node can be a subnetwork of several nodes which turn out to form local receptive fields and (linear or nonlinear) pooling.

Thus, the ELM-LRF network will be invariant to minor translations and rotations. Moreover, the connections between the input and combinatorial nodes may learn the local structures even better: denser connections around the input node because of the overlap of three receptive fields (see Fig. 3) while sparser farther away. Averaging and max-pooling methods are straightforward to formulate these combinatorial nodes. Additionally, learning-based method is also potential to construct them.

IV. The Implementation of Local Receptive Fields

A. Specific Combinatorial Node of ELM-LRF

Although different types of local receptive fields and combinatorial nodes can be used in ELM, for the convenience of implementation, we use the simple step probability function as the

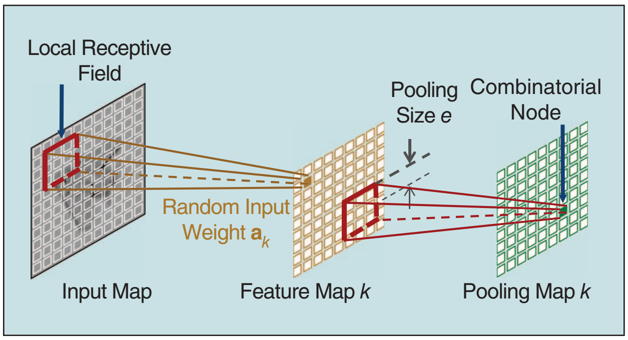


FIGURE 4 Although ELM-LRF supports wide type of local receptive fields as long as they are nonlinear piecewise continuous, a random convolutional node can be considered as a specific combinatorial node of ELM-LRF.

sampling distribution and the square/square-root pooling structure to form the combinatorial node. The receptive field of each hidden node will be composed of input nodes within a predetermined distance to the center. Furthermore, simply sharing the input weights to different hidden nodes directly leads to the convolution operation and can be easily implemented.

In this way, we build a specific case (Fig. 4) for the general ELM-LRF:

1) *Random* convolutional node in the feature map in Fig. 4 is a type of the locally connected hidden node in Fig. 3.
2) Node in the pooling map in Fig. 4 is an example of the combinatorial node shown in Fig. 3.

B. Random Input Weights

In order to obtain thorough representations of the input, K different input weights will be adopted and K diverse feature maps are generated. And Fig. 5 depicts the implementation network we use with all K maps in this paper.

In Fig. 5, hidden layer is composed of random convolutional nodes. And the input weights of the same feature maps are shared while distinct among different maps. The input weights are randomly generated and then orthogonalized as follows:

1) Generate the initial weight matrix $\hat{\mathbf{A}}^{\text{init}}$ randomly[1]. With the input size $d \times d$ and receptive field $r \times r$, the size of the feature map should be $(d-r+1) \times (d-r+1)$.

$$\begin{aligned} &\hat{\mathbf{A}}^{\text{init}} \in \mathbf{R}^{r^2 \times K} \\ &\hat{\mathbf{a}}_k^{\text{init}} \in \mathbf{R}^{r^2}, k = 1, \cdots, K \end{aligned} \tag{17}$$

2) Orthogonalize the initial weight matrix $\hat{\mathbf{A}}^{\text{init}}$ using singular value decomposition (SVD) method. Columns of $\hat{\mathbf{A}}$, $\hat{\mathbf{a}}_k$'s, are the orthonormal basis of $\hat{\mathbf{A}}^{\text{init}}$[2].

The orthogonalization allows the network to extract a more complete set of features than non-orthogonal ones. Hence, the generalization performance of the network is further improved. Orthogonal random weights have also been used in [5], [57] and produce great results.

The input weight to the k-th feature map is $\mathbf{a}_k \in \mathbf{R}^{r \times r}$, which corresponds to $\hat{\mathbf{a}}_k \in \mathbf{R}^{r^2}$ column-wisely. The convolutional node (i, j) in the k-th feature map, $c_{i,j,k}$, is calculated as:

$$\begin{aligned} &c_{i,j,k}(\mathbf{x}) = \sum_{m=1}^{r} \sum_{n=1}^{r} x_{i+m-1,\, j+n-1} \cdot a_{m,n,k} \\ &i,j = 1, \cdots, (d-r+1) \end{aligned} \tag{18}$$

Several works have demonstrated that CNN with certain structure is also able to present surprisingly good performance with ELM feature mapping (through nonlinear mapping after random input weights [22], [58], [59]). However, the performance in these previous works are worse than fine-tuned ones. In this paper, the proposed ELM-LRF, where the input weights

[1]In this specific random convolutional node case, we find that the bias is not required in the hidden nodes. And we generate the random input weights from the standard Gaussian distribution.

[2]In the case $r^2 < K$, orthogonalization cannot be performed on the column. Instead, the approximation method is used: 1) $\hat{\mathbf{A}}^{\text{init}}$ is transposed; 2) orthogonalized; 3) transposed back.

to the feature maps are orthogonal random, can achieve even better performance than well-trained counterpart. Additionally the relationship between ELM-LRF and CNN will be discussed later.

C. Square/Square-Root Pooling Structure

The square/square-root pooling structure is used to formulate the combinatorial node. Pooling size e is the distance between the center and the edge of the pooling area as shown in Fig. 5. And the pooling map is of the same size with the feature map $(d-r+1)\times(d-r+1)$. $c_{i,j,k}$ and $h_{p,q,k}$ respectively denote node (i,j) in the k-th feature map and combinatorial node (p,q) in the k-th pooling map.

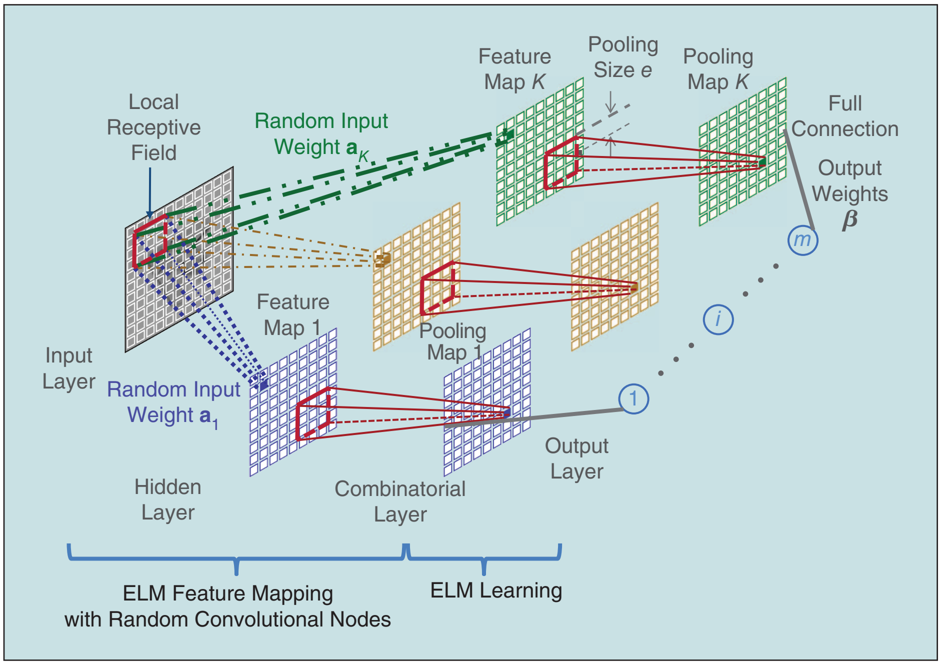


FIGURE 5 The implementation network of ELM-LRF with K maps.

$$h_{p,q,k} = \sqrt{\sum_{i=(p-e)}^{p+e}\sum_{j=(q-e)}^{q+e} c_{i,j,k}^2}$$
$$p,q = 1,\cdots,(d-r+1)$$
$$\text{if } (i,j) \text{ is out of bound: } c_{i,j,k} = 0 \tag{19}$$

The square and summation operations respectively introduce rectification nonlinearity and translational invariance into the network, which have been discovered to be the essential factors for successful image processing and similar tasks [60]. Furthermore, the square/square-root pooling structure followed convolution operation has been proved to be frequency selective and translational invariant [58]. Therefore, the implementation network we use in this paper will be especially suitable for image processing.

D. Closed-Form Solution Based Output Weights

The pooling layer is in full connection with the output layer as shown in Fig. 5. The output weights β are analytically calculated as in the unified ELM [12] using regularized least-squares method [61].

For one input sample $\mathbf{x}$, the value of combinatorial node $h_{p,q,k}$ is obtained by calculating (18) and (19) cascadingly:

$$c_{i,j,k}(\mathbf{x}) = \sum_{m=1}^{r}\sum_{n=1}^{r} x_{i+m-1,\, j+n-1}\cdot a_{m,n,k}$$
$$h_{p,q,k} = \sqrt{\sum_{i=(p-e)}^{p+e}\sum_{j=(q-e)}^{q+e} c_{i,j,k}^2}$$
$$p,q = 1,\cdots,(d-r+1)$$
$$\text{if } (i,j) \text{ is out of bound: } c_{i,j,k} = 0 \tag{20}$$

Simply concatenating the values of all combinatorial nodes into a row vector and putting the rows of N input samples together, we obtain the combinatorial layer matrix $\mathbf{H} \in \mathbf{R}^{N\times K\cdot(d-r+1)^2}$:

1) if $N \le K\cdot(d-r+1)^2$

$$\boldsymbol{\beta} = \mathbf{H}^T\left(\frac{\mathbf{I}}{C}+\mathbf{H}\mathbf{H}^T\right)^{-1}\mathbf{T} \tag{21}$$

2) if $N > K\cdot(d-r+1)^2$

$$\boldsymbol{\beta} = \left(\frac{\mathbf{I}}{C}+\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{T} \tag{22}$$

V. Discussions

A. Universal Approximation and Classification Capability

ELM-LRF with local randomly connected hidden nodes can be regarded as a specific type of ELM:

1) Different density of network connections can be built according to some type of continuous probability distribution. According to ELM theories (e.g., Theorems 2.1 and 2.2), such networks still have universal approximation and classification capabilities.
2) Disconnected input nodes and hidden nodes can be considered as connected by links with insignificant weights which can be neglected while dense connections are randomly generated with some type of continuous probability distribution. Overall speaking, the probability distribution which is used to generate such connections is still continuous, and thus, ELM with such local receptive fields still remains its universal approximation and classification capabilities.
3) Moreover, hidden nodes in ELM can be the combination of different computational nodes in linear or nonlinear manner, which naturally results in local receptive fields.

As those hidden nodes are usually nonliner piecewise continuous as stated in Theorems 2.1 and 2.2, their combinatorial node (p,q) in the k-th pooling map $h_{p,q,k}$ can still generally be expressed as the basic form of ELM hidden node (2):

$$h_{p,q,k} = G(\mathbf{a}_{p,q}, b_{p,q}, \mathbf{x}), \ \ p,q = 1,\cdots,(d-r+1) \tag{23}$$

Apparently the function G provided by the square/squareroot pooling structure is nonlinear piecewise continuous. Therefore, the universal approximation and classification capabilities are preserved for ELM-LRF, enabling it to learn sophisticated forms of input data.

B. Relationships with HTM and CNN

The implementation of ELM-LRF shares some resemblance with HTM [25] in the sense that through constructing the learning model layer-by-layer, they both mimic the brain to deal with increasingly complex forms of input. However, ELM-LRF is more efficient as the connections and input weights are both randomly generated while HTM might require to carefully design the network and tune the parameters.

In addition, the proposed ELM-LRF is also closely related to CNN. They both handle the raw input directly and apply local connections to force the network to learn spatial correlations in natural images and languages. Subsequently, high-level features are implicitly learned or generated, on which the learning is subsequently performed. However, several differences exist between ELM-LRF and CNN:

1) Local receptive fields: ELM-LRF provides more flexible and wider type of local receptive fields, which could be sampled randomly based on different types of probability distributions. However, CNN only uses convolutional hidden nodes. Although this paper simply chooses random convolutional nodes as a specific local receptive field of ELM-LRF, it is worth investigating other suitable local receptive fields and learning methods to formulate the combinatorial nodes as the local receptive fields in biological learning (e.g. brains) may be unknown but their learning capabilities have been guaranteed by ELM theories.
2) Training: Hidden nodes in CNN need to be tuned and BP method is usually adopted. Thus, CNN faces the trivial issues associated with BP, such as local minima and slow convergence rate. In contrast, ELM-LRF randomly generates the input weights and analytically calculates the output weights. Only the calculation of output weights involves major computations, making ELM-LRF deterministic and efficient.

ELM feature mapping has also been used in some CNN network implementation (with random convolutional weights) [58]. However, the feature maps and pooling maps are collectively used as the input to a support vector machine (SVM) for learning [58]. Our proposed ELM-LRF simply uses a genuine ELM architecture with combinatorial nodes as hidden nodes in ELM feature mapping stage while keeping the rest ELM learning mechanism unchanged.

VI. Experiments

The proposed ELM-LRF has been compared with state-of-the-art deep learning algorithms on NORB object recognition dataset [62], which is often used as a benchmark database by deep learning community. NORB dataset contains 24300 stereo images for training and 24300 stereo images for testing, each belonging to 5 generic categories with many variabilities such as 3D pose and lighting. Fig. 6 displays some examples from the dataset. Each sample has 2 images (left and right sides) with normalized object size and uniform background. We downsize them to 32×32 without any other preprocessing.

The experimental platform is MATLAB 2013a, Intel Xeon E5-2650, 2GHz CPU, 256GB RAM. In ELM-LRF, parameters that need to be tuned include: 1) size of receptive field $\{4\times4, 6\times6\}$; 2) number of feature maps $\{24, 36, 48, 60\}$; 3) pooling size $\{1, 2, 3, 4\}$; 4) value of C $\{0.01, 0.1, 1, 10, 100\}$. We use a hold-out validation set to choose the optimal parameters and list them in Table 1.

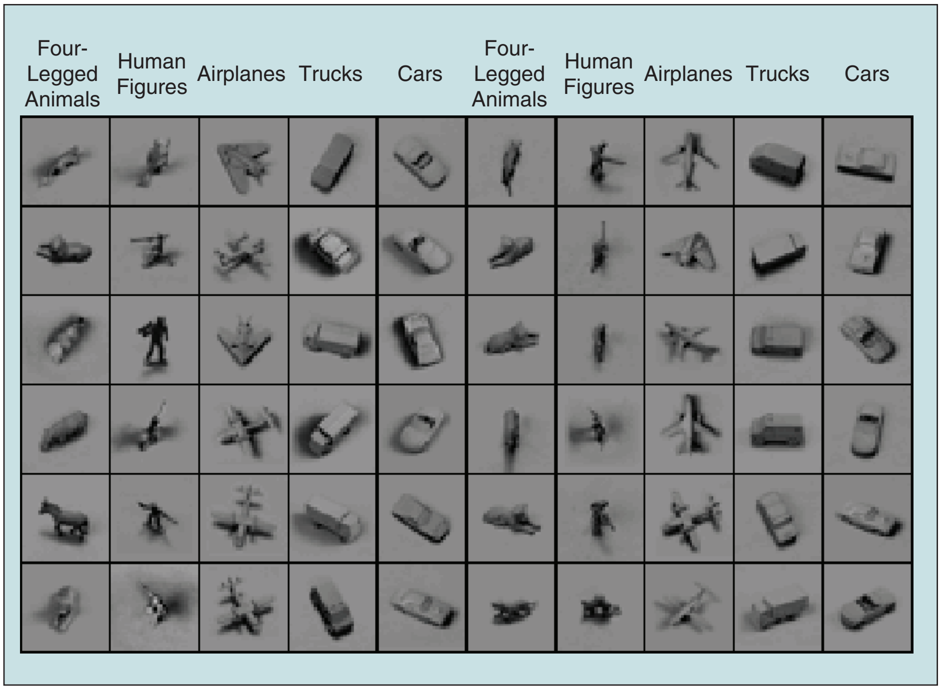


FIGURE 6 60 samples of NORB dataset.

TABLE 1 Optimal parameters.

DATASET	# OF TRAINING DATA	# OF TESTING DATA	INPUT DIMENSIONS	RECEPTIVE FIELD	# OF FEATURE MAPS	POOLING SIZE	C
NORB	24300	24300	$32 \times 32 \times 2$	4×4	48	3	0.01

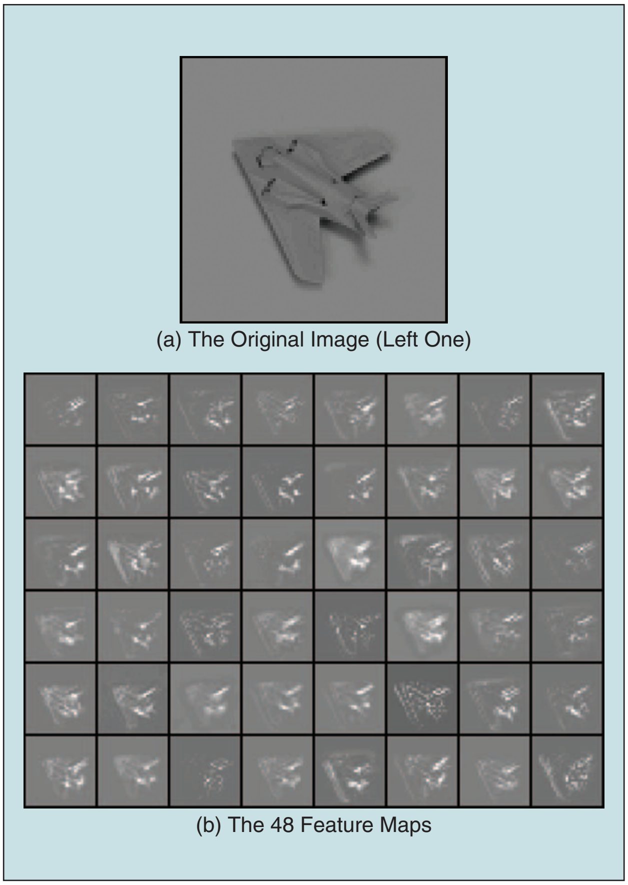
TABLE 2 Test error rates of different algorithms.

ALGORITHMS	TEST ERROR RATES
ELM-LRF	**2.74%**
ELM-LRF (NO ORTHOGONALIZATION)	4.01%
RANDOM WEIGHTS [58] (ELM FEATURE MAPPING + SVM CLASSIFIER)	4.8%
K-MEANS + SOFT ACTIVATION [60]	2.8%
TILED CNN [57]	3.9%
CNN [62]	6.6%
DBN [63]	6.5%

TABLE 3 Training time comparison.

ALGORITHMS	TRAINING TIME (S)	SPEEDUP TIMES[1]
ELM-LRF	394.16	217.47
ELM-LRF (NO ORTHOGONALIZATION)	391.89	218.73
RANDOM WEIGHTS[2] (ELM FEATURE MAPPING + SVM CLASSIFIER)	1764.28	48.58
K-MEANS + SOFT ACTIVATION[3]	6920.47	12.39
TILED CNN[4]	15104.55	5.67
CNN[5]	53378.16	1.61
DBN[6]	85717.14	1

[1]DBN IS USED AS THE STANDARD TO CALCULATE THE SPEEDUP TIMES.
[2]THE LATEST FASTEST CNN SOLUTION. THE TRAINING TIME REPORTED IN [58] IS 0.1H. REASONS FOR SUCH DIFFERENCE IN TRAINING TIME ARE: I) WE HAVE CONSIDERED CONVOLUTION AND POOLING OPERATIONS AS PART OF TRAINING, AND THE TRAINING TIME REPORTED INCLUDES THE TIME SPENT ON CONVOLUTION AND POOLING, WHICH WERE NOT CONSIDERED IN [58]; II) EXPERIMENTS ARE RUN IN DIFFERENT EXPERIMENTAL PLATFORMS.
[3]USE THE SAME PARAMETERS AS IN [60] WITH 4000 FEATURES.
[4]USE THE SAME ARCHITECTURE AS ELM-LRF AND SET THE MAXIMUM NUMBER OF ITERATIONS IN THE PRETRAINING TO 20.
[5]THE ARCHITECTURE IS PROVIDED IN [62] AND WE USE 50 EPOCHS.
[6]THE ARCHITECTURE IS: 2048 (INPUT)-500-2000-500-5 (OUTPUT) AND 500 EPOCHS SINCE THE CONVERGENCE OF TRAINING ERROR IS SLOW WHEN DEALING WITH NORB DATASET.



(a) The Original Image (Left One)

(b) The 48 Feature Maps

FIGURE 7 One sample: the original image (left) and corresponding feature maps.

A. Test Error

The test errors of other algorithms have been reported in literature and summarized in Table 2[3]. ELM-LRF produces better accuracy than other fine-tuned algorithms that requires much more computations. Compared with the conventional deep learning solutions, CNN and DBN, the proposed ELM-LRF reduces the test error rate from 6.5% to 2.74%. Our proposed ELM-LRF achieves the *best accuracy* compared to those results reported in literature with a significant gap.

B. Training Time

For the training time, other algorithms are reproduced on our experimental platform to conduct a fair comparison. As shown in Table 3, ELM-LRF learns up to *200 times* faster than other algorithms. And even compared with the random weights method [58], which is the current most efficient algorithm, ELM-LRF still achieves 4.48 times speedup and reaches much higher testing accuracy.

C. Feature Maps

Fig. 7 uses one data sample (airplane, the left image) for illustration. And it also displays the corresponding 48 feature maps generated in the hidden layer. It can be observed that the outlines of these feature maps are similar, as they all are generated from the same input image (an airplane). However, each map has its distinct highlighted part such that diverse representations of the original image are obtained. Collectively, these feature maps represent different abstractions of the original image, making the subsequent classification easy and accurate.

D. Orthogonalization of Random Input Weights

In our experiments, we have also investigated the contribution of orthogonalization after the random generation of input weights. The values of the center convolutional nodes in the 48 feature maps are taken for example. The value distributions generated by orthogonal and non-orthogonal random weights are shown in Fig. 8. One data sample is picked from each generic category. As observed from Fig. 8, the distribution of orthogonal random weights spreads more broadly and evenly. It is similar for convolutional nodes occupying other positions in the feature maps. Therefore, orthogonalization will make the objects more easily to be linearly separated and classified.

In addition, even without orthogonalization, ELM-LRF can still achieve 4.01% test error, which is 38% decrease comparing to conventional deep learning methods, as shown in Table 2. Moreover, if considering the superb efficiency, ELM-LRF (no

[3]Although we tried to reproduce the results of other algorithms, we find that it is hard to reach similar accuracy as reported in literature due to tedious parameters tuning and time-consuming human intervention in conventional deep learning methods.

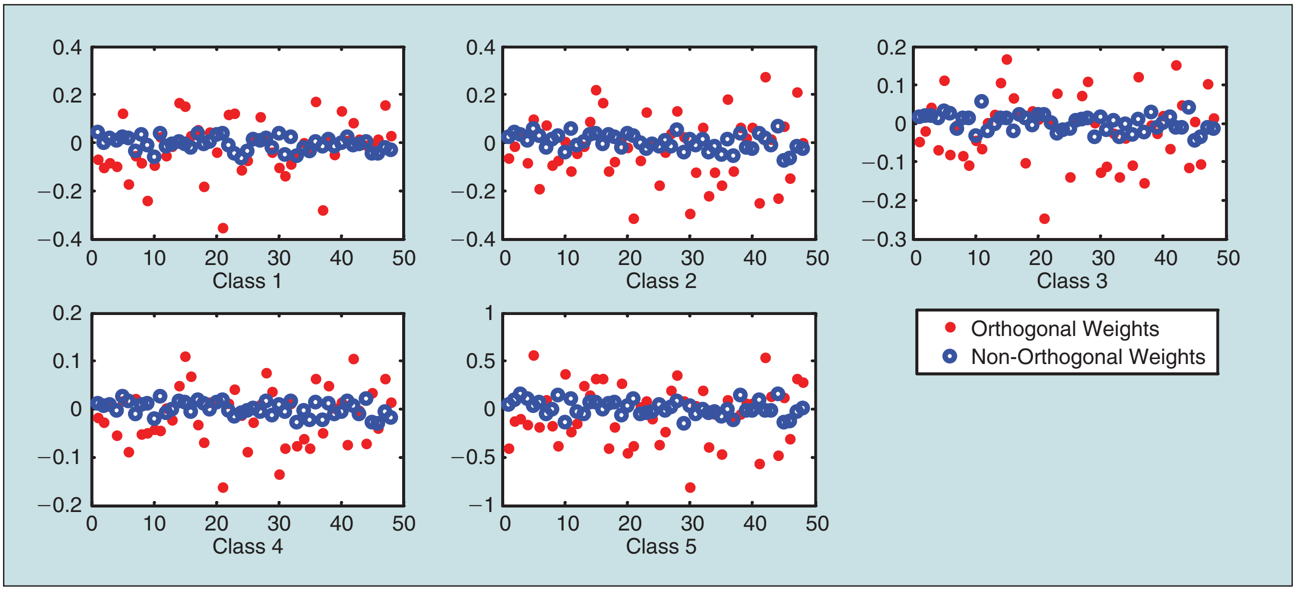


FIGURE 8 The value distributions of the center convolutional nodes in the 48 feature maps with orthogonal and non-orthogonal random weights.

orthogonalization) is also an excellent solution for image processing and similar applications.

VII. Conclusions

This paper studies the local receptive fields of ELM (ELM-LRF) which naturally results from the randomness of hidden neurons and combinatorial hidden nodes suggested in ELM theories. According to ELM theories, different types of local receptive fields can be used and there exists close relationship between local receptive fields and random hidden neurons. The local connections between input and hidden nodes may allow the network to consider local structures and combinatorial nodes further introduce translational invariance into the network. Input weights are randomly generated and then orthogonalized in order to encourage a more complete set of features. The output weights are calculated analytically in ELM-LRF, providing a simple deterministic solution.

In the implementation, we construct a specific network for simplicity using the step function to sample the local connections and the square/square-root pooling structure to formulate the combinatorial nodes. Surprisingly random convolutional nodes can be considered an efficient implementation of local receptive fields of ELM although many different types of local receptive fields are supported by ELM theories. Experiments conducted on the benchmark object recognition task, NORB dataset, show that the proposed ELM-LRF presents the *best accuracy* in the literature and accelerate the training speed up to *200 times* compared to popular deep learning algorithms.

ELM theories show that different local receptive fields in ELM-LRF (and biological learning mechanisms) may be generated due to different probability distribution used in generating random hidden neurons. It may be worth investigating in the future: 1) the impact of different ELM local receptive fields; 2) the impact of different combinatorial hidden nodes of ELM; 3) stacked network of ELM-LRF modules performing unsupervised learning on higher-level abstractions.

Strictly speaking, from network architecture point of view, this paper simply uses one hidden layer of ELM although a combinatorial hidden layer is used. Stacked networks have been extensively investigated, such as stacked autoencoders (SAE), deep belief network (DBN) and convolutional DBN [63]–[66]. It deserves further exploration to construct the stacked network of ELM-LRF, by performing subsequent local connections on the output of the previous combinatorial layer. The stacked network will capture and learn higher-level abstractions. Thus, it should be able to deal with even more complex forms of input. In addition, unsupervised learning method, such as ELM Auto-Encoder (ELM-AE) [5], might be helpful for the stacked network to obtain more abstract representations without losing salient information.

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References

[1] C. P. Chen and C.-Y. Zhang, "Data-intensive applications, challenges, techniques and technologies: A survey on big data," *Inform. Sci.*, vol. 275, pp. 314–347, Aug. 2014.

[2] Z.-H. Zhou, N. Chawla, Y. Jin, and G. Williams, "Big data opportunities and challenges: Discussions from data analytics perspectives," *IEEE Comput. Intell. Mag.*, vol. 9, pp. 62–74, Nov. 2014.

[3] Y. Zhai, Y.-S. Ong, and I. Tsang, "The emerging 'big dimensionality,'" *IEEE Comput. Intell. Mag.*, vol. 9, pp. 14–26, Aug. 2014.

[4] I. Arnaldo, K. Veeramachaneni, A. Song, and U. O'Reilly, "Bring your own learner: A cloud-based, data-parallel commons for machine learning," *IEEE Comput. Intell. Mag.*, vol. 10, pp. 20–32, Feb. 2015.

[5] L. L. C. Kasun, H. Zhou, G.-B. Huang, and C. M. Vong, "Representational learning with extreme learning machine for big data," *IEEE Intell. Syst.*, vol. 28, no. 6, pp. 31–34, 2013.

[6] D. E. Rumelhart, G. E. Hinton, and R. J. Williams, "Learning representations by back-propagation errors," *Nature*, vol. 323, pp. 533–536, Oct. 1986.

[7] G.-B. Huang, Q.-Y. Zhu, and C.-K. Siew, "Extreme learning machine: A new learning scheme of feedforward neural networks," in *Proc. Int. Joint Conf. Neural Networks*, July 2004, vol. 2, pp. 985–990.

[8] G.-B. Huang, Q.-Y. Zhu, and C.-K. Siew, "Extreme learning machine: Theory and applications," *Neurocomputing*, vol. 70, pp. 489–501, Dec. 2006.
[9] G.-B. Huang, L. Chen, and C.-K. Siew, "Universal approximation using incremental constructive feedforward networks with random hidden nodes," *IEEE Trans. Neural Networks*, vol. 17, no. 4, pp. 879–892, 2006.
[10] G.-B. Huang and L. Chen, "Convex incremental extreme learning machine," *Neurocomputing*, vol. 70, pp. 3056–3062, Oct. 2007.
[11] G.-B. Huang and L. Chen, "Enhanced random search based incremental extreme learning machine," *Neurocomputing*, vol. 71, pp. 3460–3468, Oct. 2008.
[12] G.-B. Huang, H. Zhou, X. Ding, and R. Zhang, "Extreme learning machine for regression and multiclass classification," *IEEE Trans. Syst. Man, Cybern. B*, vol. 42, no. 2, pp. 513–529, 2012.
[13] G.-B. Huang, "An insight into extreme learning machines: Random neurons, random features and kernels," *Cognitive Comput.*, vol. 6, no. 3, pp. 376–390, 2014.
[14] Z.-H. You, Y.-K. Lei, L. Zhu, J. Xia, and B. Wang, "Prediction of protein-protein interactions from amino acid sequences with ensemble extreme learning machines and principal component analysis," *BMC Bioinform.*, vol. 14, suppl. 8, p. S10, 2013.
[15] Y. Song, J. Crowcroft, and J. Zhang, "Automatic epileptic seizure detection in EEGs based on optimized sample entropy and extreme learning machine," *J. Neurosci. Methods*, vol. 210, no. 2, pp. 132–146, 2012.
[16] Y. Zhang and P. Zhang, "Optimization of nonlinear process based on sequential extreme learning machine," *Chem. Eng. Sci.*, vol. 66, no. 20, pp. 4702–4710, 2011.
[17] Y. Yang, Y. Wang, and X. Yuan, "Bidirectional extreme learning machine for regression problem and its learning effectiveness," *IEEE Trans. Neural Networks Learning Syst.*, vol. 23, no. 9, pp. 1498–1505, 2012.
[18] Z. Yan and J. Wang, "Robust model predictive control of nonlinear systems with unmodeled dynamics and bounded uncertainties based on neural networks," *IEEE Trans. Neural Networks Learning Syst.*, vol. 25, no. 3, pp. 457–469, 2014.
[19] A. Nizar, Z. Dong, and Y. Wang, "Power utility nontechnical loss analysis with extreme learning machine method," *IEEE Trans. Power Syst.*, vol. 23, no. 3, pp. 946–955, 2008.
[20] R. Minhas, A. A. Mohammed, and Q. M. J. Wu, "Incremental learning in human action recognition based on snippets," *IEEE Trans. Circuits Syst. Video Technol.*, vol. 22, no. 11, pp. 1529–1541, 2012.
[21] Y. Zhou, J. Peng, and C. L. P. Chen, "Extreme learning machine with composite kernels for hyperspectral image classification," *IEEE J. Select. Topics Appl. Earth Observ. Remote Sensing*, 2014, to be published.
[22] K. Jarrett, K. Kavukcuoglu, M. Ranzato, and Y. LeCun, "What is the best multi-stage architecture for object recognition?" in *Proc. IEEE Int. Conf. Computer Vision*, 2009, pp. 2146–2153.
[23] Z. Bai, G.-B. Huang, D. Wang, H. Wang, and M. B. Westover, "Sparse extreme learning machine for classification," *IEEE Trans. Cybern.*, vol. 44, no. 10, pp. 1858–1870, 2014.
[24] S. Lawrence, C. L. Giles, A. C. Tsoi, and A. D. Back, "Face recognition: A convolutional neural-network approach," *IEEE Trans. Neural Networks*, vol. 8, no. 1, pp. 98–113, 1997.
[25] J. Hawkins and S. Blakeslee, *On Intelligence*. New York: Macmillan, 2007.
[26] G. Huang, G.-B. Huang, S. Song, and K. You, "Trends in extreme learning machines: A review," *Neural Netw.*, vol. 61, no. 1, pp. 32–48, 2015.
[27] Y.-H. Pao, G.-H. Park, and D. J. Sobajic, "Learning and generalization characteristics of the random vector functional-link net," *Neurocomputing*, vol. 6, pp. 163–180, Apr. 1994.
[28] C. P. Chen, "A rapid supervised learning neural network for function interpolation and approximation," *IEEE Trans. Neural Networks*, vol. 7, no. 5, pp. 1220–1230, 1996.
[29] C. Chen, S. R. LeClair, and Y.-H. Pao, "An incremental adaptive implementation of functional-link processing for function approximation, time-series prediction, and system identification," *Neurocomputing*, vol. 18, no. 1, pp. 11–31, 1998.
[30] C. P. Chen and J. Z. Wan, "A rapid learning and dynamic stepwise updating algorithm for flat neural networks and the application to time-series prediction," *IEEE Trans. Syst. Man, Cybern. B*, vol. 29, no. 1, pp. 62–72, 1999.
[31] Y.-H. Pao, *Adaptive Pattern Recognition and Neural Networks*. Boston, MA: Addison-Wesley Longman Publishing Co., Inc., 1989.
[32] B. Igelnik and Y.-H. Pao, "Stochastic choice of basis functions in adaptive function approximation and the functional-link net," *IEEE Trans. Neural Networks*, vol. 6, no. 6, pp. 1320–1329, 1995.
[33] H. White, "An additional hidden unit test for neglected nonlinearity in multilayer feedforward networks," in *Proc. Int. Conf. Neural Networks*, 1989, pp. 451–455.
[34] T.-H. Lee, H. White, and C. W. J. Granger, "Testing for neglected nonlinearity in time series models: A comparison of neural network methods and standard tests," *J. Econometrics*, vol. 56, pp. 269–290, 1993.
[35] M. B. Stinchcombe and H. White, "Consistent specification testing with nuisance parameters present only under the alternative," *Econometric Theory*, vol. 14, no. 3, pp. 295–324, 1998.
[36] H. White, "Approximate nonlinear forecasting methods," in *Handbook of Economics Forecasting*, G. Elliott, C. W. J. Granger, and A. Timmermann, Eds. New York: Elsevier, 2006, pp. 460–512.
[37] G. Huang, S. Song, J. N. Gupta, and C. Wu, "Semi-supervised and unsupervised extreme learning machines," *IEEE Trans. Cybern.*, vol. 44, no. 12, pp. 2405–2417, 2014.
[38] A. Rahimi and B. Recht, "Uniform approximation of functions with random bases," in *Proc. 46th Annu. Allerton Conf. Communication, Control, Computing*, Sept. 2008, pp. 555–561.
[39] G.-B. Huang, Q.-Y. Zhu, K. Z. Mao, C.-K. Siew, P. Saratchandran, and N. Sundararajan, "Can threshold networks be trained directly?" *IEEE Trans. Circuits Syst. II*, vol. 53, no. 3, pp. 187–191, 2006.
[40] W. F. Schmidt, M. A. Kraaijveld, and R. P. Duin, "Feedforward neural networks with random weights," in *Proc. 11th IAPR Int. Conf. Pattern Recognition Methodology Systems*, 1992, pp. 1–4.
[41] B. Widrow, A. Greenblatt, Y. Kim, and D. Park, "The no-prop algorithm: A new learning algorithm for multilayer neural networks," *Neural Netw.*, vol. 37, pp. 182–188, Jan. 2013.
[42] G.-B. Huang, X. Ding, and H. Zhou, "Optimization method based extreme learning machine for classification," *Neurocomputing*, vol. 74, pp. 155–163, Dec. 2010.
[43] C. R. Rao and S. K. Mitra, *Generalized Inverse of Matrices and Its Applications*. vol. 7, New York: Wiley, 1971.
[44] B. Frénay and M. Verleysen, "Using SVMs with randomised feature spaces: An extreme learning approach," in *Proc. 18th European Symp. Artificial Neural Networks*, Apr. 28–30, 2010, pp. 315–320.
[45] C. Cortes and V. Vapnik, "Support vector networks," *Mach. Learn.*, vol. 20, no. 3, pp. 273–297, 1995.
[46] T. Poggio, S. Mukherjee, R. Rifkin, A. Rakhlin, and A. Verri, "b," A.I. Memo No. 2001-011, CBCL Memo 198, Artificial Intelligence Laboratory, Massachusetts Institute of Technology, Cambridge, MA, 2001.
[47] I. Steinwart, D. Hush, and C. Scovel, "Training SVMs without offset," *J. Machine Learning Res.*, vol. 12, pp. 141–202, Feb. 2011.
[48] P. L. Bartlett, "The sample complexity of pattern classification with neural networks: The size of the weights is more important than the size of the network," *IEEE Trans. Inform. Theory*, vol. 44, no. 2, pp. 525–536, 1998.
[49] D. H. Hubel and T. N. Wiesel, "Receptive fields and functional architecture of monkey striate cortex," *J. Physiol.*, vol. 195, no. 1, pp. 215–243, 1968.
[50] D. Scherer, A. Müller, and S. Behnke, "Evaluation of pooling operations in convolutional architectures for object recognition," in *Proc. Int. Conf. Artificial Neural Networks*, 2010, pp. 92–101.
[51] M. Pal, A. E. Maxwell, and T. A. Warner, "Kernel-based extreme learning machine for remote-sensing image classification," *Remote Sensing Lett.*, vol. 4, no. 9, pp. 853–862, 2013.
[52] J. Butcher, D. Verstraeten, B. Schrauwen, C. Day, and P. Haycock, "Reservoir computing and extreme learning machines for non-linear time-series data analysis," *Neural Netw.*, vol. 38, pp. 76–89, Feb. 2013.
[53] W. Zheng, Y. Qian, and H. Lu, "Text categorization based on regularization extreme learning machine," *Neural Comput. Applicat.*, vol. 22, nos. 3–4, pp. 447–456, 2013.
[54] P. Kara and R. C. Reid, "Efficacy of retinal spikes in driving cortical responses," *J. Neurosci.*, vol. 23, no. 24, pp. 8547–8557, 2003.
[55] D. L. Sosulski, M. L. Bloom, T. Cutforth, R. Axel, and S. R. Datta, "Distinct representations of olfactory information in different cortical centres," *Nature*, vol. 472, no. 7342, pp. 213–216, 2011.
[56] O. Barak, M. Rigotti, and S. Fusi, "The sparseness of mixed selectivity neurons controls the generalization-discrimination trade-off," *J. Neurosci.*, vol. 33, no. 9, pp. 3844–3856, 2013.
[57] Q. V. Le, J. Ngiam, Z. Chen, D. Chia, P. W. Koh, and A. Y. Ng, "Tiled convolutional neural networks," in *Proc. Advances Neural Information Processing Systems*, 2010, pp. 1279–1287.
[58] A. Saxe, P. W. Koh, Z. Chen, M. Bhand, B. Suresh, and A. Y. Ng, "On random weights and unsupervised feature learning," in *Proc. Int. Conf. Machine Learning*, 2011, pp. 1089–1096.
[59] D. Cox and N. Pinto, "Beyond simple features: A large-scale feature search approach to unconstrained face recognition," in *Proc. IEEE Int. Conf. Automatic Face Gesture Recognition Workshops*, 2011, pp. 8–15.
[60] A. Coates, A. Y. Ng, and H. Lee, "An analysis of single-layer networks in unsupervised feature learning," in *Proc. Int. Conf. Artificial Intelligence Statistics*, 2011, pp. 215–223.
[61] D. W. Marquaridt, "Generalized inverses, ridge regression, biased linear estimation, and nonlinear estimation," *Technometrics*, vol. 12, no. 3, pp. 591–612, 1970.
[62] Y. LeCun, F. J. Huang, and L. Bottou, "Learning methods for generic object recognition with invariance to pose and lighting," in *Proc. Int. Conf. Computer Vision Pattern Recognition*, July 2004, vol. 2, pp. II-97–104.
[63] V. Nair and G. E. Hinton, "3D object recognition with deep belief nets," in *Proc. Advances Neural Information Processing Systems*, 2009, pp. 1339–1347.
[64] Y. Bengio, "Learning deep architectures for AI," *Foundations Trends R Machine Learning*, vol. 2, no. 1, pp. 1–127, 2009.
[65] M. A. Ranzato, C. Poultney, S. Chopra, and Y. LeCun, "Efficient learning of sparse representations with an energy-based model," in *Proc. Advances Neural Information Processing Systems*, 2006, pp. 1137–1144.
[66] H. Lee, R. Grosse, R. Ranganath, and A. Y. Ng, "Convolutional deep belief networks for scalable unsupervised learning of hierarchical representations," in *Proc. Int. Conf. Machine Learning*, 2009, pp. 609–616.